

$$\begin{aligned} \textcircled{1} \quad & X_1 - X_2 + 2X_3 + 3X_4 = -1 \\ & 0 + X_2 - 4X_3 + X_4 = 3 \\ & 2X_1 + X_2 + 2X_3 + X_4 = 9 \\ & X_1 + 0 + 0 - 2X_4 = 6 \end{aligned}$$

Solving by substitution and elimination..

Eqn 2 & 3 have same values of X_2 , get rid of X_4

Eqn 4 + eqn 2

$$(2X_1 + X_2 + 2X_3 + X_4 = 9) - (0 + X_2 - 4X_3 + X_4 = 3)$$

$$2X_1 - 2X_3 = 3$$

eqn 3 - eqn 1

add eqn 1 & 2

$$(X_1 - X_2 + 2X_3 + 3X_4 = -1) + (0 + X_2 - 4X_3 + X_4 = 3)$$

$$X_1 - 2X_3 + 4X_4 = 2$$

we have 3 equations

$$2X_1 - 2X_3 = 3$$

$$X_1 - 2X_4 = 6$$

$$X_1 - 2X_3 + 4X_4 = 2$$

from $2X_1 - 2X_3 = 3$;

$$-2X_3 = 3 - 2X_1$$

$$X_3 = -\frac{3}{2} + X_1$$

$$X_1 - 2\left(-\frac{3}{2} + X_1\right) + 4X_4 = 2$$

$$\cancel{X_1 - 2}$$

$$-X_1 + 4X_4 = -1 \Rightarrow X_1 - 4X_4 = 1$$

we now have 2 equations

Continuation of question I

$$X_1 - 2X_4 = 6$$

$$X_1 - 4X_4 = 1$$

$$\rightarrow X_1 = 6 + 2X_4$$

$$\rightarrow 6 + 2X_4 - 4X_4 = 1$$

$$6 - 2X_4 = 1$$

$$2X_4 = 5$$

$$X_4 = \frac{5}{2}$$

$$X_1 = 6 + 2\left(\frac{5}{2}\right) \Rightarrow X_1 = 6 + 5 = 11$$

$$X_3 = -\frac{3}{2} + X_1 \Rightarrow -\frac{3}{2} + 11 = \frac{19}{2}$$

$$X_2 = 3 + 4X_4 - X_1$$

$$X_2 = 3 + 4\left(\frac{5}{2}\right) - 11 = 3 + 10 - 11 = 2$$

$$X_2 = \frac{77}{2}$$

$$X_1 = 11$$

$$X_2 = \frac{77}{2}$$

$$X_3 = \frac{19}{2}$$

$$X_4 = \frac{5}{2}$$

$$\begin{bmatrix} 1 & -2 & 1 & -1 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 1 & 0 & 3 & 2 & 0 \end{bmatrix}$$

Question 2

Add a dummy equation

$$\begin{bmatrix} 1 & -2 & 1 & -1 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 1 & 0 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solving for echelon

$$\begin{array}{l} -2R_2/R_1 \\ R_1 \end{array} \begin{array}{c} x_1 \quad x_2 \quad x_3 \quad x_4 \\ \left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 0 \\ 4 & 1 & -2 & 2 & 0 \\ 1 & 0 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \end{array}$$

$$R_3 + R_2 \quad \left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 0 \\ 4 & 1 & -2 & 2 & 0 \\ 5 & 1 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$-R_1 - R_4 \quad \left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & 0 \\ 4 & 1 & -2 & 2 & 0 \\ 5 & 1 & 1 & 4 & 0 \\ -1 & 2 & -1 & 1 & 0 \end{array} \right]$$

$$X_1 = 0$$

$$X_2 = 0$$

$$X_3 = 0$$

$$X_4 = 0$$

the equations can not be solved further.

$$X_3 + X_2$$

Q13b)

$$X_1 - 2X_2 = 2$$

$$X_1 + 4X_2 = 8$$

$$X_1 + X_2 = 5$$

$$\left[\begin{array}{cc|c} 1 & -2 & 2 \\ 1 & 4 & 8 \\ 1 & 1 & 5 \end{array} \right]$$

$$\begin{array}{l} R_2 - R_1 \\ R_3 - R_1 \end{array} \left[\begin{array}{cc|c} 1 & -2 & 2 \\ 0 & 6 & 6 \\ 0 & 3 & 3 \end{array} \right]$$

$$\begin{array}{l} R_2/6 \\ R_3/3 \end{array} \left[\begin{array}{cc|c} 1 & -2 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{array} \right]$$

$$R_3 - R_2 \left[\begin{array}{cc|c} 1 & -2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array} \right]$$

$$R_1 + 2R_2 \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array} \right]$$

$$\Rightarrow X_1 = 4$$

$$\Rightarrow X_2 = 1$$

Question 3a)

2 equations with 3 Variables

$$2x + 3y + 4z = 10$$

$$4x + 6y + 8z = 3$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 4 & 10 \\ 4 & 6 & 8 & 3 \end{array} \right]$$

$$R_2/R_1 \left[\begin{array}{ccc|c} 2 & 3 & 4 & 10 \\ 2 & 3 & 4 & 3/10 \end{array} \right]$$

$$R_2 - R_1 \left[\begin{array}{ccc|c} 2 & 3 & 4 & 10 \\ 0 & 0 & 0 & -97/10 \end{array} \right]$$

the equation is inconsistent. It has no equation.